Grade 4 Mathematics, Quarter 2, Unit 2.1

Adding and Subtracting Fractions and Mixed Numbers

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b* (e.g. ¾ is equivalent to ¼ + ¼ + ¼).
- Build fractions from unit fractions with a common denominator.
- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole (of a set or area).
- Decompose fractions into sum of fractions with the same denominator in more than one way Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
- Rename whole numbers as a fraction with a like denominator.
- Understand a mixed number is a whole number plus a fraction smaller than one written without the plus sign (e.g. 7 1/5 = 7 + 1/5).
- Rename mixed numbers as an equivalent fraction.
- Add and subtract mixed numbers with like denominators by renaming mixed numbers as equivalent fractions or by using properties of operations (example of renaming: 1 1/3 + 2/3 = 4/3 + 2/3 = 6/3; or example of using properties of operations: 1 1/3 + 2/3 = 1 + (1/3 + 2/3) = 1 + 1 = 2).
- Solve word problems involving addition and subtraction of fractions referring to the same whole and like denominators by using visual models and equations to represent problems.

Mathematical practices to be integrated

Model with mathematics.

- Simplify the situation to a simpler rule, allowing for revisions.
- Draw conclusions, interpret results, and revise models if needed.

Essential questions

- What does the numerator of a fraction represent? What does the denominator tell you?
- What is a unit fraction?
- How can you decompose a fraction and how would you record it as an equation in a variety of different ways?
- What does a mixed number represent?
- How do you add or subtract with like denominators?
- What is your strategy for converting mixed numbers and improper fractions?
- What is your strategy for renaming a mixed number?
- How you can add or subtract mixed numbers with like denominators using equivalent fractions and/or properties of operations? Why does your strategy make sense?
- What visual fraction model and its related equation would you use to represent this problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] 4.NF

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students gained an understanding of fractions $1/b$ as the quantity formed by 1 part when a whole is divided into b equal parts. They understand a fraction a/b as the quantity formed by a parts of size $1/b$. Students represented fractions on a number line. Additionally, students partitioned shapes into

parts with equal areas and expressed them as a fraction of the whole. An understanding of equivalence was developed and students generated equivalent fractions. They expressed whole numbers as fractions and compared fractions with like denominators. Students learned that shapes have various attributes and that they can be grouped into categories.

Current Learning

In grade 4, fractions are one of the three critical areas. Fractions are a first priority of instruction according to the PARCC documents. At a developmental level, students will understand a fraction *a/b* with $a > 1$ as a sum of fractions $1/b$. Furthermore, they gain an understanding of addition and subtraction of fractions as joining and separating parts referring to the same whole. Students decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation (e.g., $3/8 = 1/8 + 1/8 + 1/8$). They add and subtract mixed numbers with like denominators, and solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

Students should understand that a mixed number is simply the addition of a whole number added to a fraction smaller than 1 and be able to represent a mixed number as a fraction with a like denominator. Converting between mixed numbers and improper fractions should not be a rote procedure, but rather an application of their understanding of composing/decomposing fractions with like denominators. It is important to recognize that these concepts are at the developmental level, but progressing toward reinforcement. In a future unit, fourth-graders will work with multiplying fractions and whole numbers.

Future Learning

In fifth grade, students will be adding and subtracting fractions with unlike denominators, including mixed numbers. Students will be interpreting fractions as division of the numerator by the denominator. They will be interpreting the product as a partition of equal parts of a whole. Additionally, students will be solving real world problems involving multiplication of fractions and mixed numbers.

Additional Findings

According to *Principles and Standards for School Mathematics*, during grades 3–5, students should build their understanding of fractions as parts of a whole and as division. They will need to see and explore a variety of models of fractions focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions. They should develop strategies for ordering and comparing fractions often using benchmarks such as 1/2 and 1 (p. 150).

The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating their sums can be different. Just as the sum of 4 and 7 can be seen as the length of the segment obtained by joining together two segments of lengths 4 and 7, so the sum of 2/3 and 8/5. It is not necessary to know how much $2/3 + 8/5$ is exactly in order to know what the sum means. This is analogous to understanding 51 x 78 as 51 groups of 78, without necessarily knowing its exact value (*Progressions for the Common Core State Standards in Mathematics: 3-5 Number and Operations—Fractions,* p. 6).

Grade 4 Mathematics, Quarter 2, Unit 2.2

Developing Understanding of Multidigit Multiplication

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Multiply whole numbers up to four digits by one digit.
- Multiply two-digit whole numbers by two-digit whole numbers.
- Use strategies based on place value and the properties of operations.
- Illustrate and explain calculation using equations, rectangular arrays, and/or area models.
- Multiply to solve (multistep) word problems involving multiplicative comparison.
- Represent problems using equations with a letter standing for the unknown quantity.
- Apply area and perimeter formulas for rectangles in real-world and math problems.
- Use estimation strategies to assess reasonableness of answers.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities in problem situations.
- Know and flexibly use different properties of operations and objects.

Attend to precision.

- Calculate accurately by rechecking for precision.
- Define mathematical symbols and units of measure consistently and appropriately.

Look for and express regularity in repeated reasoning.

- Look for patterns to simplify.
- Use a variety of strategies and properties to verify answers.

Essential questions

- What strategy would you use to multiply a two-digit by two-digit number?
- What strategy could you use to solve a problem where more than operation is needed?
- How can your knowledge of place value help you solve (given) problem?
- How can you use a letter to represent a number in an equation?
- What are the formula(s) you use to find the area and perimeter of a rectangle?
- How can you illustrate and explain a calculation using an equation, rectangular array, and/or area models?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply ~~or divide~~ to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

[1] See Glossary, Table 2.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers ~~using the four operations, including problems in which remainders must be interpreted~~. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In third grade, students developed an understanding of the meanings of multiplication and division of whole numbers through problem solving and modeling. They found unknown factors and used properties of operations to calculate products involving single-digit factors. Using these concepts, they solved two-step word problems using equations with a letter to represent the unknown quantity. Students have applied the properties of operations of (communicative, associative, distributive) as strategies to multiply and divide. Students have developed the concept of area measurement by counting unit squares. They have related the concepts of multiplication and addition to find the area of a rectangle with whole-number side lengths. They have used area models to represent the distributive property.

Current Learning

In grade 4, the development of understanding and fluency with multidigit multiplication is a critical area of instruction according to the CCSS. Students use their understanding of place value to multiply a whole number of up to four digits by a one-digit whole number. They also multiply two two-digit numbers using their knowledge of place value at the developmental level. They illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Students multiply to solve problems involving multiplicative comparison (e.g., using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison).

Future Learning

In fifth grade, students will finalize fluency with multidigit addition, subtraction, multiplication, and division. They will apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Further meaning for multiplication should develop as students build and describe area models showing how a product is related to its factors. The area model is important because it helps students develop an understanding of multiplication properties. Other relationships can be seen by decomposing and composing area models. Using area models, properties of operations, such as commutativity of multiplication, become more apparent. The distributive property is particularly powerful as the basis of many efficient multiplication algorithms" (p. 165).

When students work toward meeting this standard, they combine prior understanding of multiplication when deepening understanding of the base-ten system of units to express the product of two multidigit numbers as another multidigit number. This work will continue in grade 5 and culminate in fluency with the standard algorithms in grade 6. (*PARCC Model Content Frameworks: Mathematics* Grade 3–11, p. 20.)

Grade 4 Mathematics, Quarter 2, Unit 2.3

Developing Understanding of Division with Remainders

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies.
- Illustrate and explain calculations using equations, rectangular arrays and/or area models.
- Solve multistep word problems involving whole numbers (all 4 operations) including problems with remainders that must be interpreted.
- Represent problems using equations with a letter representing the unknown quantity.
- Use mental computation and estimation strategies (including rounding) to check reasonableness of answers.

Mathematical practices to be integrated

Model with mathematics.

- Solve problems in everyday life through the use of algorithms.
- Identify important quantities and their relationships and express them as an equation.

Attend to precision.

- Calculate accurately by rechecking for precision.
- Define mathematical symbols and units of measure consistently and appropriately.

Look for and make use of structure.

- Use a variety of strategies and properties to verify their answers.
- Break numbers apart.

Essential questions

- In context, explain what the remainder represents.
- How is multiplication useful in solving division problems?
- What is your strategy for solving problems with multiple operations?
- How do you show a model of a division problem (equations, rectangular arrays, and/or area models)?
- Why is estimation useful?
- How do you estimate?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students learned to interpret the number of objects in each share when they are partitioned equally. They used multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities. Students also fluently divided numbers through 100 using strategies like the relationship between multiplication and division. It is important to note that the focus in third grade was to develop an understanding of multiplication and division simply as an *unknown factor* problem. Finally, in grade 3, students used area models as part of the process of understanding multiplication and division.

Current Learning

In grade 4, the development of understanding and fluency with multi-digit multiplication is a critical area of instruction according to the CCSS. According to the PARCC Model Content Frameworks, using their understanding of place value and properties of operations to perform multi-digit arithmetic is a major cluster. Students find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Remainders are interpreted in respect to context. (Example problem: *Suppose 250 pencils were distributed among 9 students with each student receiving the same number of pencils. If the teacher got the remaining pencils, how many pencils did the teacher get?*). Students use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division at the developmental level. They illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

In fourth grade, students multiply or divide to solve word problems involving multiplicative comparison (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison). Furthermore, students solve multistep word problems posed with whole numbers and having whole number answers using each operation, including problems in which remainders must be interpreted. Students will represent these problems using equations with a letter standing for the unknown quantity. They will also assess the reasonableness of answers using mental computation and estimation strategies including rounding. It is important to note that some of these concepts are reinforcement, but the division skills are at the developmental level.

Future Learning

In fifth grade, students will continue working with division. They will find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors using strategies based on place value. They will use the properties of operations and or the relationship between multiplication and division. Students will continue to illustrate and explain the calculation by using equations, rectangular arrays and/or area models. Students will begin to use parentheses, brackets, or braces in numerical expressions and then evaluate expressions with those symbols. They will be writing simple expressions that record calculations with numbers and interpreting numerical expressions without evaluating them.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Further meaning for multiplication should develop as students build and describe area models showing how a product is related to its factors. The area model is important because it helps students develop an understanding of multiplication properties. Other relationships can be seen by decomposing and composing area models. Using area models, properties of operations such as commutativity of multiplication become more apparent. The distributive property is particularly powerful as the basis of many efficient multiplication algorithms" (p. 165).

When students work toward meeting this standard, they combine prior understanding of multiplication when deepening understanding of the base-ten system of units to express the product of two multi-digit numbers as another multi-digit number. This work will continue in grade 5 and culminate in fluency with the standard algorithms in grade 6. (*PARCC Model Content Frameworks: Mathematics Grade 3-11,* p. 20.)

In problem situations, students must interpret and use remainders with respect to context (*Progressions: K-5 Operations and algebraic Thinking*, p. 30)

According to the *PARCC Content Model Frameworks*, when students work toward meeting 4.NBT.5 standard, they combine prior understanding of multiplication and division with deepening understanding of the base ten systems of units to find whole number quotients with remainders with up to 4 digit dividend and one digit divisors. This work will develop further in grade 5 and culminate in fluency with the standard algorithm in grade 6 (heading: Examples of Opportunities for In Depth focus, p. 20).